BENHA UNIVERSITY
FACULTY OF ENGINEERING AT SHOUBRA

ECE-312
Electronic Circuits (A)

Lecture # 5
Hybrid Equivalent Model

Instructor:
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NOVEMBER 2014

Agenda

- Two-Port Systems Approach
- Cascaded Systems
- The Hybrid Equivalent Model
- Approximate & Complete h-model
- Hybrid π Model
- Troubleshooting and Practical Applications

TWO PORT SYSTEMS APPROACH

2-Port System

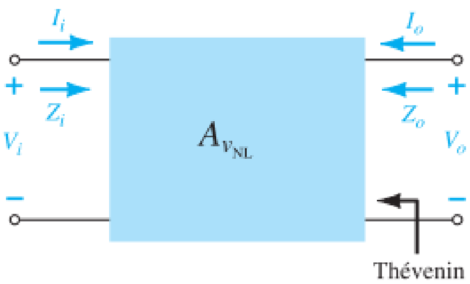


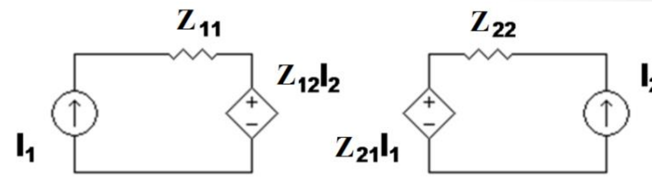
FIG. 5.61
Two-port system.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where

$$z_{11} \overset{\text{def}}{=} \left. \frac{V_1}{I_1} \right|_{I_2=0} \qquad z_{12} \overset{\text{def}}{=} \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

$$z_{21} \overset{\text{def}}{=} \left. \frac{V_2}{I_1} \right|_{I_2=0} \qquad z_{22} \overset{\text{def}}{=} \left. \frac{V_2}{I_2} \right|_{I_1=0}$$



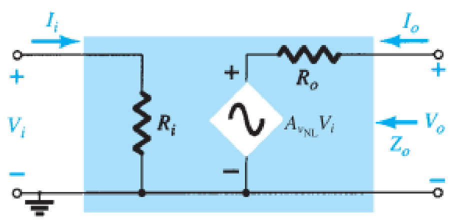


FIG. 5.62
Substituting the internal elements for the two-port system of Fig. 5.61.

$$V_o = A_{v_{NL}} V_i$$

$$Z_o = R_o \qquad Z_i = R_i$$

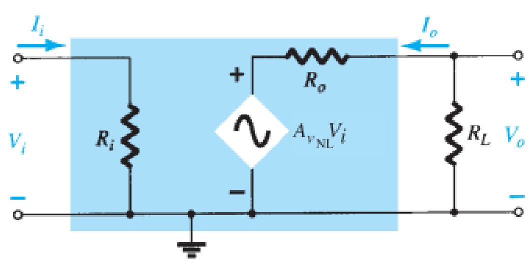


FIG. 5.63
Applying a load to the two-port system of Fig. 5.62.

$$V_o = \frac{R_L A_{v_{NL}} V_i}{R_L + R_o}$$

$$A_{v_L} = \frac{V_o}{V_i} = \frac{R_L}{R_L + R_o} A_{v_{NL}}$$

$$A_{i_L} = \frac{I_o}{I_i} = \frac{-V_o/R_L}{V_i/Z_i} = -\frac{V_o}{V_i}\frac{Z_i}{R_L}$$

$$A_{i_L} = -A_{v_L}\frac{Z_i}{R_L}$$

2-Port System..

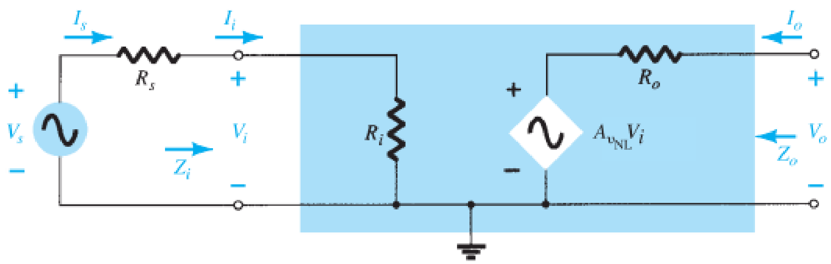


FIG. 5.64
Including the effects of the source resistance R_s.

$$V_i = \frac{R_i V_s}{R_i + R_s}$$

$$V_o = A_{v_{NL}} V_i$$

$$V_o = A_{v_{NL}} \frac{R_i}{R_i + R_s} V_s$$

$$A_{v_s} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} A_{v_{NL}}$$

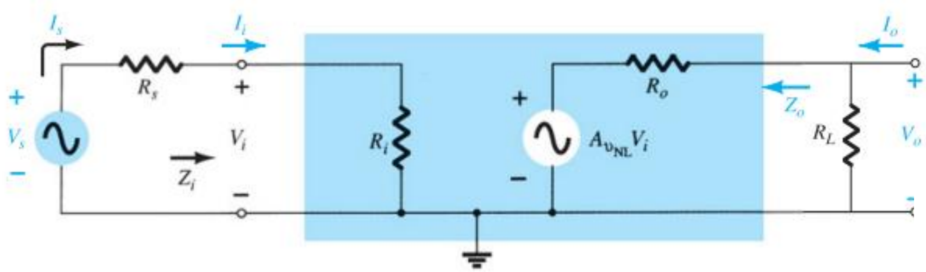


FIG. 5.65
Considering the effects of R_s and R_L on the gain of an amplifier.

$$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s}$$

$$V_o = \frac{R_L}{R_L + R_o} A_{v_{NL}} V_i$$

$$A_{v_L} = \frac{V_o}{V_i} = \frac{R_L A_{v_{NL}}}{R_L + R_o} = \frac{R_L}{R_L + R_o} A_{v_{NL}}$$

$$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_i}{V_s}$$

$$A_{v_s} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} \cdot \frac{R_L}{R_L + R_o} A_{v_{NL}}$$

$$A_{i_L} = -A_{v_L} \frac{R_i}{R_L}$$

$$A_{i_s} = -A_{v_s} \frac{R_s + R_i}{R_L}$$

CASCADED SYSTEMS

Cascaded Systems

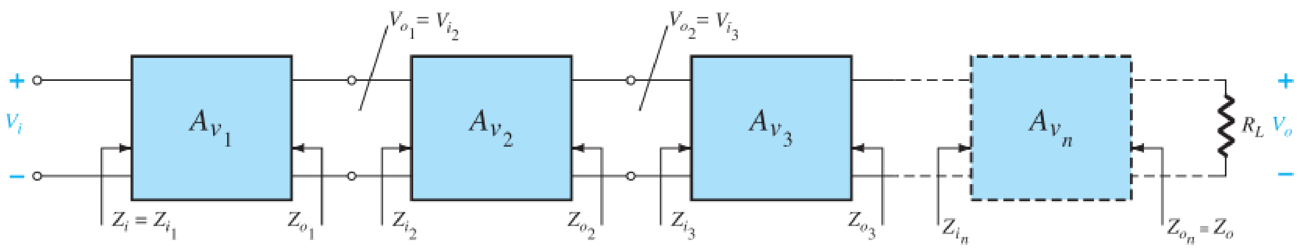
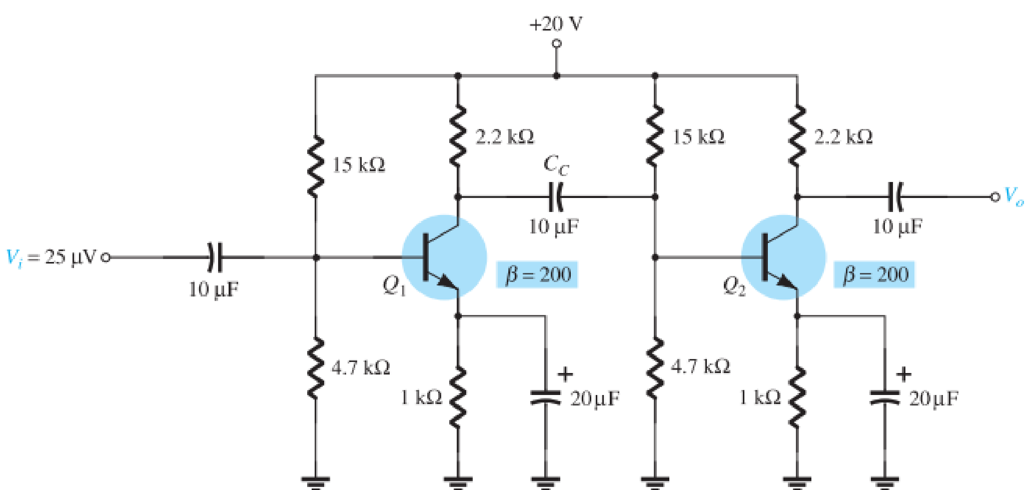


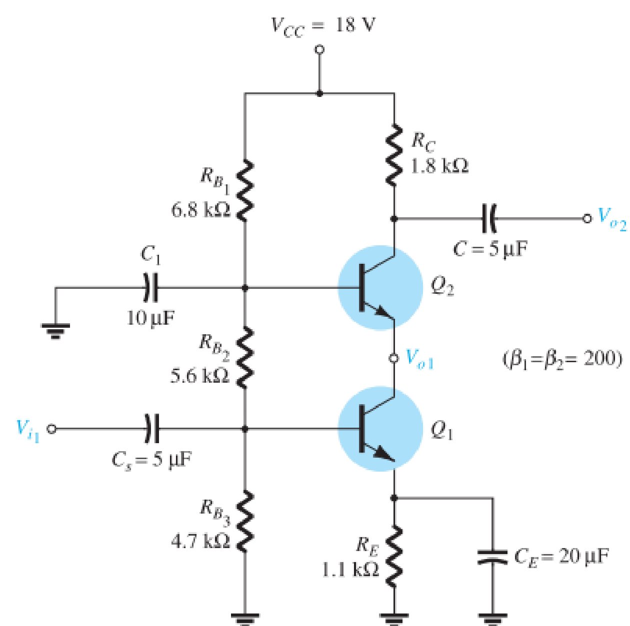
FIG. 5.67
Cascaded system.

$$A_{v_T} = A_{v_1} \cdot A_{v_2} \cdot A_{v_3} \cdots$$

$$A_{i_T} = -A_{v_T} \frac{Z_{i_1}}{R_L}$$

- Examples: RC Coupled ct & Cascode ct
- Check Examples: 5.15 & 5.16





THE HYBRID EQUIVALENT MODEL

The Hybrid Equivalent Model

- The r_e model has the advantage that the parameters are defined by the actual operating conditions,
- the parameters of the hybrid equivalent circuit are defined in general terms for any operating conditions.

		Min.	Max.	
Input impedance ($I_C = 1$ mA dc, $V_{CE} = 10$ V dc, $f = 1$ kHz)	h_{ie}	0.5	7.5	kΩ
Voltage feedback ratio ($I_C = 1$ mA dc, $V_{CE} = 10$ V dc, $f = 1$ kHz)	h_{re}	0.1	8.0	$\times 10^{-4}$
Small-signal current gain ($I_C = 1$ mA dc, $V_{CE} = 10$ V dc, $f = 1$ kHz)	h_{fe}	20	250	—
Output admittance ($I_C = 1$ mA dc, $V_{CE} = 10$ V dc, $f = 1$ kHz)	h_{oe}	1.0	30	μS

FIG. 5.92
Hybrid parameters for the 2N4400 transistor.

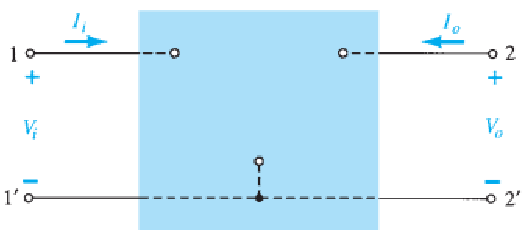


FIG. 5.93
Two-port system.

$$V_i = h_{11}I_i + h_{12}V_o$$

$$I_o = h_{21}I_i + h_{22}V_o$$

$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_o=0}$$ ohms → short-circuit input-impedance parameter

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i=0}$$ unitless → open-circuit reverse transfer voltage ratio parameter

$$h_{21} = \left.\frac{I_o}{I_i}\right|_{V_o=0}$$ unitless → short-circuit forward transfer current ratio parameter

$$h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i=0}$$ siemens → short-circuit forward transfer current ratio parameter

Transistor Hybrid Equivalent ct

$$V_i = h_{11}I_i + h_{12}V_o$$

$$I_o = h_{21}I_i + h_{22}V_o$$

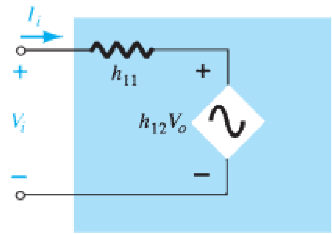


FIG. 5.94
Hybrid input equivalent circuit.

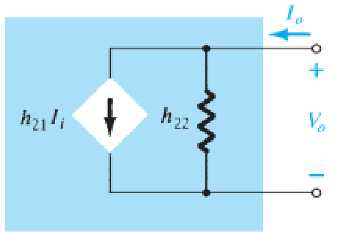


FIG. 5.95
Hybrid output equivalent circuit.

- For Transistor:

$h_{11} \rightarrow$ input resistance $\rightarrow h_i$

$h_{12} \rightarrow$ reverse transfer voltage ratio $\rightarrow h_r$

$h_{21} \rightarrow$ forward transfer current ratio $\rightarrow h_f$

$h_{22} \rightarrow$ output conductance $\rightarrow h_o$

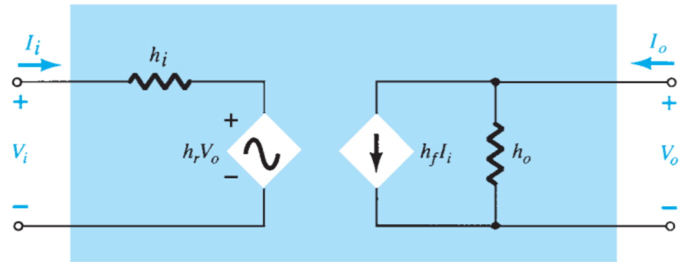


FIG. 5.96
Complete hybrid equivalent circuit.

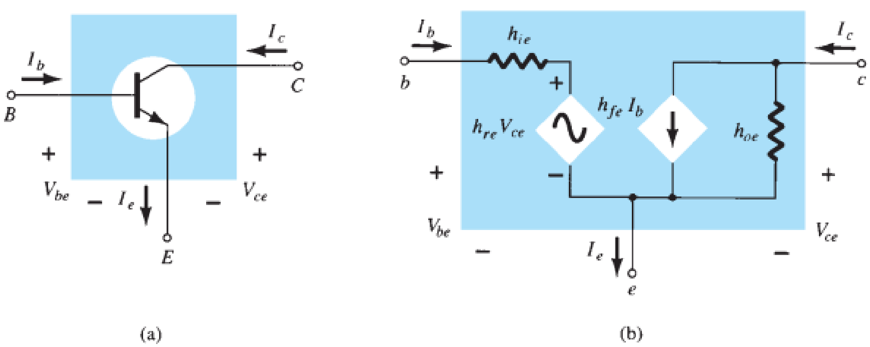


FIG. 5.97
Common-emitter configuration: (a) graphical symbol; (b) hybrid equivalent circuit.

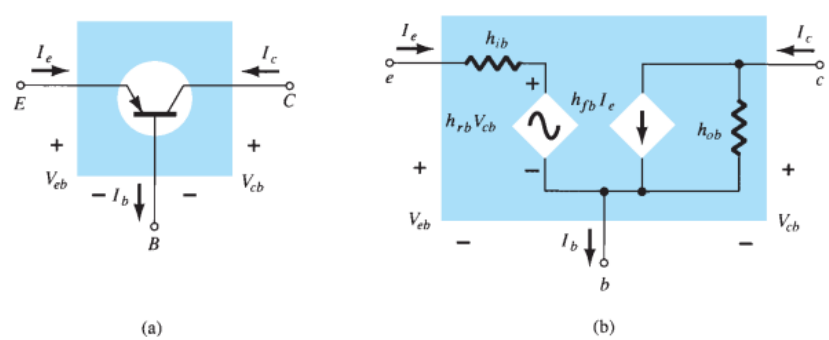


FIG. 5.98
Common-base configuration: (a) graphical symbol; (b) hybrid equivalent circuit.

Hybrid vs. r_e model

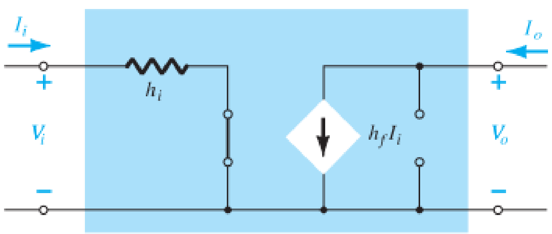


FIG. 5.99
Effect of removing h_{re} and h_{oe} from the hybird equivalent circuit.

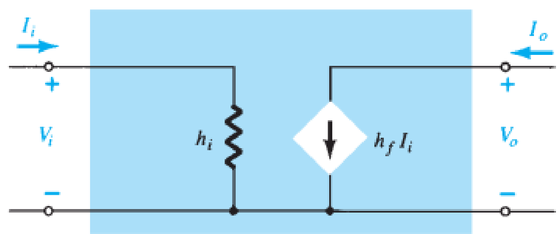


FIG. 5.100
Approximate hybrid equivalent model.

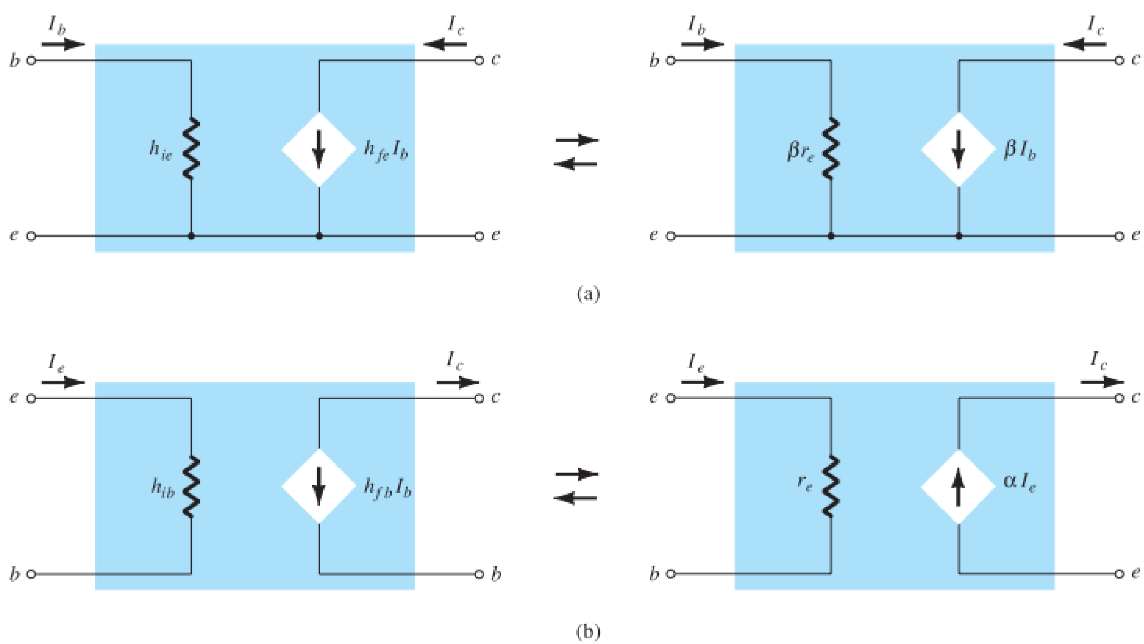


FIG. 5.101
Hybrid versus r_e model: (a) common-emitter configuration; (b) common-base configuration.

$h_{ie} = \beta r_e$

$h_{fe} = \beta_{ac}$

$h_{ib} = r_e$

$h_{fb} = -\alpha \cong -1$

APPROXIMATE & COMPLETE H-MODEL

Approximate h-model

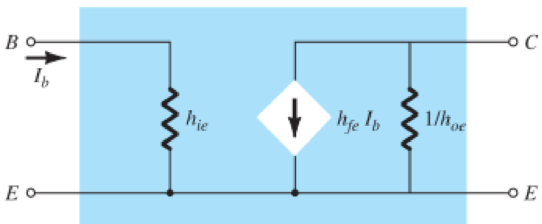


FIG. 5.104
Approximate common-emitter hybrid equivalent circuit.

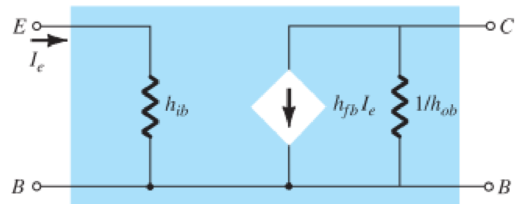


FIG. 5.105
Approximate common-base hybrid equivalent circuit.

- Fixed Bias ct

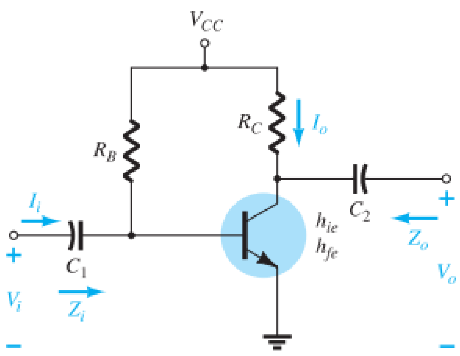


FIG. 5.106
Fixed-bias configuration.

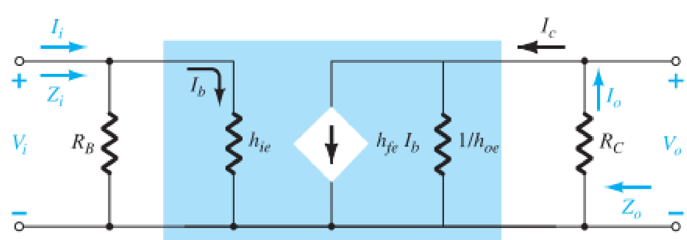


FIG. 5.107
Substituting the approximate hybrid equivalent circuit into the ac equivalent network of Fig. 5.106.

$$Z_i = R_B \| h_{ie}$$

$$Z_o = R_C \| 1/h_{oe}$$

$$R' = 1/h_{oe} \| R_C$$

$$V_o = -I_o R' = -I_C R' = -h_{fe} I_b R'$$

$$I_b = \frac{V_i}{h_{ie}}$$

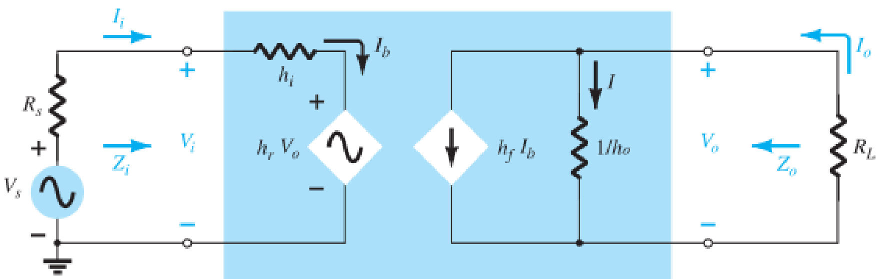
$$V_o = -h_{fe} \frac{V_i}{h_{ie}} R'$$

$$A_v = \frac{V_o}{V_i} = -\frac{h_{ie}(R_C \| 1/h_{oe})}{h_{ie}}$$

- Check other configurations !!

$$A_i = \frac{I_o}{I_i} \cong h_{fe}$$

Complete h-model



Current Gain, $A_i = I_o/I_i$

$$I_o = h_f I_b + I = h_f I_i + \frac{V_o}{1/h_o} = h_f I_i + h_o V_o$$

Substituting $V_o = -I_o R_L$ gives

$$I_o = h_f I_i - h_o R_L I_o$$

Rewriting the equation above, we have

$$I_o + h_o R_L I_o = h_f I_i$$

and $$I_o(1 + h_o R_L) = h_f I_i$$

so that

$$\boxed{A_i = \frac{I_o}{I_i} = \frac{h_f}{1 + h_o R_L}}$$

Voltage Gain, $A_v = V_o/V_i$

$$V_i = I_i h_i + h_r V_o$$

$$I_i = (1 + h_o R_L) I_o / h_f$$

and $I_o = -V_o/R_L$

$$V_i = \frac{-(1 + h_o R_L) h_i}{h_f R_L} V_o + h_r V_o$$

$$\boxed{A_v = \frac{V_o}{V_i} = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r) R_L}}$$

Input Impedance, $Z_i = V_i/I_i$

$$V_i = h_i I_i + h_r V_o$$
$$V_o = -I_o R_L$$
$$V_i = h_i I_i - h_r R_L I_o$$
$$A_i = \frac{I_o}{I_i}$$

$$I_o = A_i I_i$$

$$V_i = h_i I_i - h_r R_L A_i I_i$$

$$Z_i = \frac{V_i}{I_i} = h_i - h_r R_L A_i \qquad A_i = \frac{h_f}{1 + h_o R_L}$$

$$\boxed{Z_i = \frac{V_i}{I_i} = h_i - \frac{h_f h_r R_L}{1 + h_o R_L}}$$

Output Impedance, $Z_o = V_o/I_o$

$$V_s = 0$$

$$I_i = -\frac{h_r V_o}{R_s + h_i}$$

$$I_o = h_f I_i + h_o V_o$$
$$= -\frac{h_f h_r V_o}{R_s + h_i} + h_o V_o$$

$$\boxed{Z_o = \frac{V_o}{I_o} = \frac{1}{h_o - [h_f h_r/(h_i + R_s)]}}$$

HYBRID Π MODEL

Hybrid π Model

It includes parameters that do not appear in the other two models primarily to provide a more accurate model for high-frequency effects.

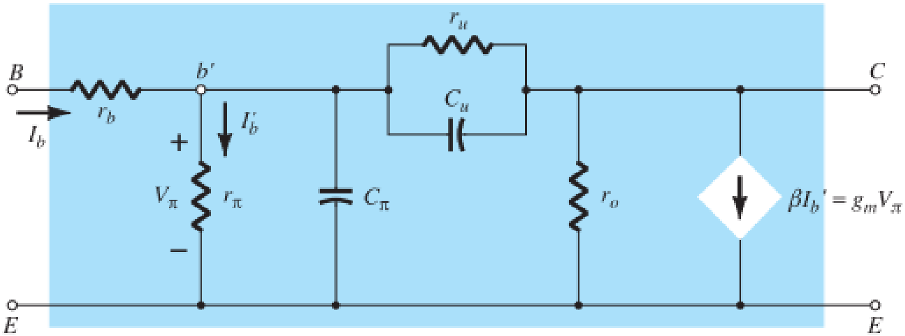


FIG. 5.123

Giacoletto (or hybrid π) high-frequency transistor small-signal ac equivalent circuit.

$$r_\pi = \beta r_e$$

$$g_m = \frac{1}{r_e}$$

$$r_o = \frac{1}{h_{oe}}$$

$$\frac{r_\pi}{r_\pi + r_u} \cong \frac{r_\pi}{r_u} \cong h_{re}$$

- The resistance r_π (using the symbol π to agree with the hybrid π terminology) is simply βr_e as introduced for the common-emitter r_e model.
- The output resistance r_o is the output resistance normally appearing across an applied load.
- The resistance r_b includes the base contact, base bulk, and base spreading resistance levels.
- The resistance r_u (the subscript u refers to the *union* it provides between collector and base terminals) is a very large resistance and provides a feedback path from output to input circuits in the equivalent model.

- All the capacitors are stray parasitic capacitors between the various junctions of the device.

- The controlled source can be a voltage-controlled current source (VCCS) or a current-controlled current source (CCCS), depending on the parameters employed.

$$\beta I_b' = \frac{1}{r_e} \cdot r_e \beta I_b' = g_m I_b' \beta r_e = g_m (I_b' r_\pi) = g_m V_\pi$$

VARIATIONS OF TRANSISTOR PARAMETERS

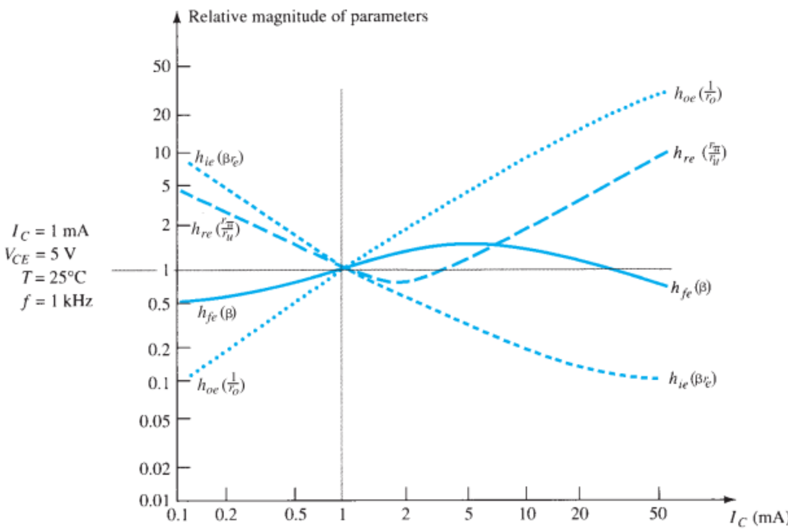


FIG. 5.124
Hybrid parameter variations with collector current.

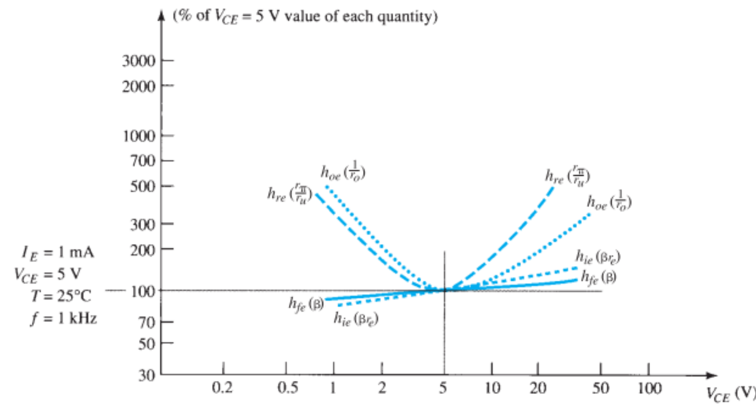


FIG. 5.125
Hybrid parameter variations with collector–emitter potential.

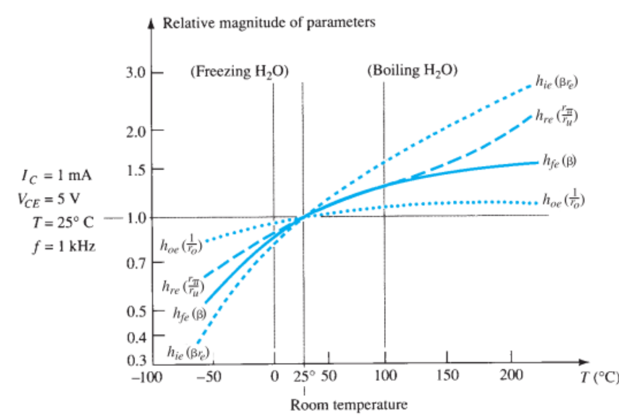


FIG. 5.126
Hybrid parameter variations with temperature.

- The parameter h_{fe}(β) varies the least of all the parameters of a transistor equivalent circuit when plotted against variations in collector current.
- All the parameters of a hybrid transistor equivalent circuit increase with temperature.

TROUBLESHOOTING & PRACTICAL APPLICATIONS

Troubleshooting

- In general, if a system is not working properly, first disconnect the ac source and check the dc biasing levels.

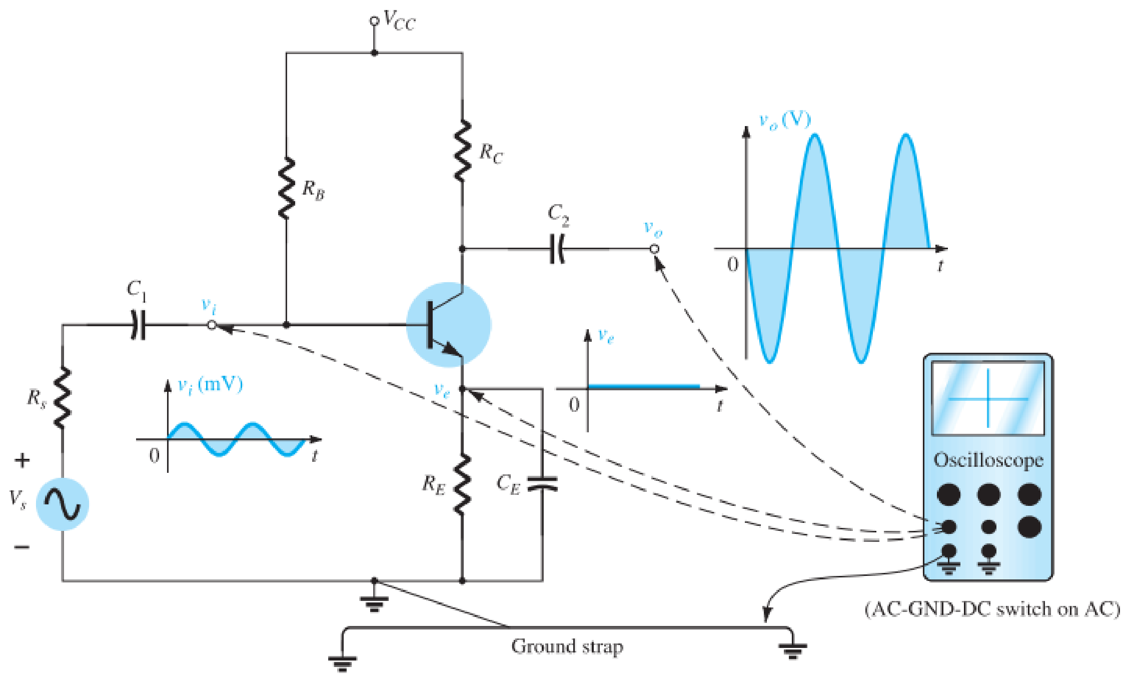


FIG. 5.128
Using the oscilloscope to measure and display various voltages of a BJT amplifier.

PRACTICAL APPLICATIONS

- Audio Mixer

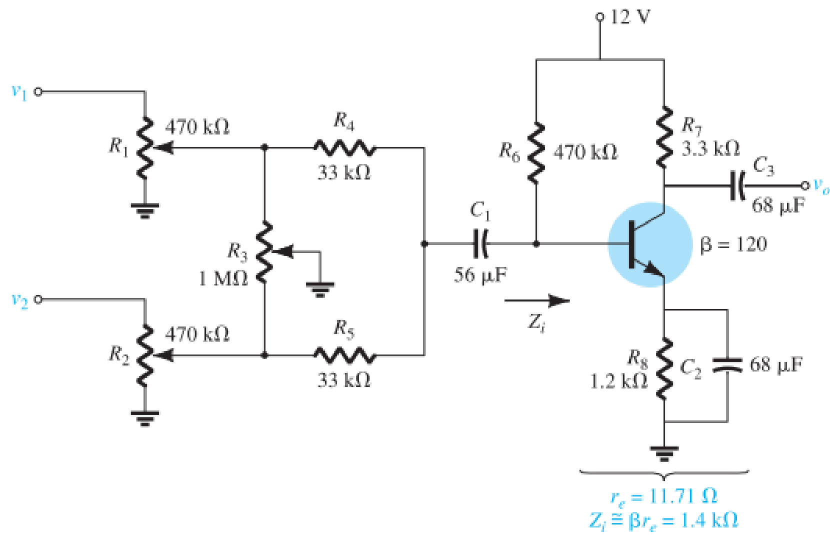


FIG. 5.130
Audio mixer.

- Preamplifier

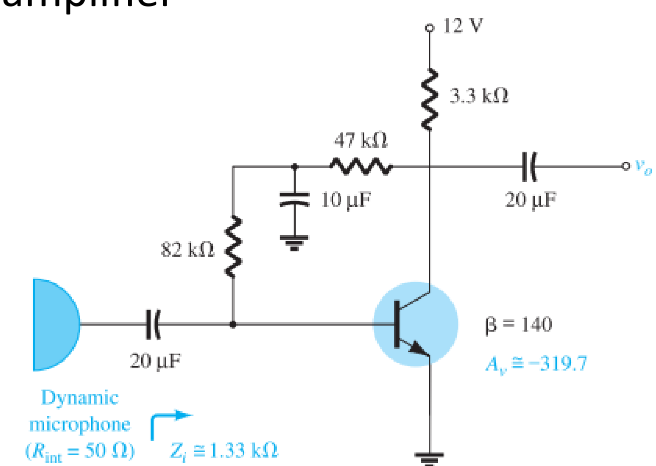


FIG. 5.133
Preamplifier for a dynamic microphone.

- Random-Noise Generator !
- Sound-Modulated Light Source !

→ Report: com3_14_R02_xx
Deadline: 17 Nov. @11:59 pm

- For more details, refer to:
  - Chapter 5 at R. Boylestad, **Electronic Devices and Circuit Theory**, 11th edition, Prentice Hall.
- The lecture is available online at:
  - http://bu.edu.eg/staff/ahmad.elbanna-courses/11966
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg